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Wu, Zeyuan (PRC-MNNU-SMS); **Chen, Hongzhang** (PRC-LAN-SSM);
Li, Jianxi (PRC-MNNU-SMS)

The size and the spectral radius of a saturated non-covered graph. (English. English summary)

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All graphs considered are simple and undirected. A connected non-empty graph G is said to be matching covered if every edge belongs to some perfect matching. A graph G is a saturated non-covered graph if it is not matching covered but $G+e$ is matching covered for every edge e in the complement of G. J. Zhou, Y. F. Cao and W. Yan [Discrete Appl. Math. **364** (2025), 53–59; MR4840996] proved that G is a saturated non-covered graph if and only if there is a set S of at least two vertices such that every $v \in S$ is adjacent to all other vertices of G, and $G-S$ contains exactly $|S|$ components, each of which is a complete graph of odd order. In this article, the authors use this structural characterization to answer some extremal questions regarding the family of saturated non-covered graphs.

Let G denote a saturated non-covered graph of even order n. First, the authors show that $e(G) \le \binom{n-1}{2}+2$ with equality if and only if G is isomorphic to $K_2 \vee (K_{n-3} \cup K_1)$ or $K_3 \vee 3K_1$, where $\vee$ denotes the join operation. Second, they show that if $n \ge \max\{\delta^3-2\delta^2+5\delta-1, 8\delta-5\}$, where δ denotes the minimum degree of G, then the spectral radius of G is at most the spectral radius of $K_\delta \vee (K_{n-2\delta+1} \cup (\delta-1)K_1)$, and the latter is the unique extremal graph for which equality is attained. Third, let the eigenvalues of the Laplacian of G be $\mu_1(G) \ge \mu_2(G) \ge \cdots \ge \mu_{n-1}(G) \ge \mu_n(G) = 0$. Then, the authors use a "separation inequality" type lemma due to X. Gu and M. H. Liu [European J. Combin. **101** (2022), Paper No. 103468; MR4338938] to show that the Laplacian eigenratio, $\mu_{n-1}(G)/\mu_1(G)$, is strictly less than 3/5.

Brahadeesh Sankarnarayanan

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